White Paper:
NIRSpec Multiplexing and the Need for Proposal Target List Over-booking

NIRSpec Instrument Science Team

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Summary: Motivated by the need to maintain flexibility in optimizing the schedule and observing efficiency of the roll-constrained JWST observatory, all JWST users will as the default be required to prepare for a range of possible telecope roll angles in their observing proposals. This requirement poses a dilemma for wide-field multi-object spectroscopy programs employing the NIRSpec Micro-Shutter Array (MSA), which are also expected to detail all targets intended to be observed in their proposals. Since the targets accessible in a NIRSpec MSA exposure cannot be identified without first knowing the precise orientation of the projection of the MSA and its grid of slits on the sky, NIRSpec MSA users will need to intentionally over-book their initial proposed target lists by some suitably large factor reflecting the possible range in roll angle for the target fields of of interest. The purpose of this note is to explore and quantify how large this over-booking needs to be – which, due to the immovable nature of the MSA shutters, turns out to be very large indeed.
Not knowing the rotational orientation of the NIRSpec field of view forces the user to consider all relevant science targets contained within an enveloping circular footprint centered on the nominal target field center, and covering an area 2.9 times larger than that covered by the four separate quadrants of the MSA at a single orientation. The finite open-area filling-factor of the shutters of the MSA array, together with the considerable number of failed shutters present in the flight device, conspire to assure that at best 32% of the randomly located targets within the MSA field of view will lie within functional three shutter long slitlets. Of these, depending on the total number of targets present in the field, typically only 20-60% will lead to spectra that can be fitted on the NIRSpec detector array without incurring overlap. Taken together, these three factors imply that maintaining roll flexibility in a NIRSpec MSA proposal requires the user to assemble and submit a target list typically containing 15-45 times more candidate targets than can actually be observed in a single MSA configuration. For on-sky primary target densities in the range $\Sigma \simeq 50-300$ arcmin^{-2} anticipated of typical NIRSpec MSA high redshift galaxy survey programs, NIRSpec will be able to achieve $45-170$ non-overlapping spectra per exposure in its workhorse $R = 100$ prism mode – well below the maximum capacity of $\simeq 197$ spectra corresponding to a fully packed detector array. Consequently, nearly all NIRSpec MSA programs will be limited in scope by the number of targets available on the sky, and are therefore statistically compelled by the late roll-assignment to submit proposal-level target lists laying claim to all known and sufficiently bright targets of interest within the roll-mandated extended field of view – in the knowledge of only being able to capture spectra of a very small fraction of these objects in a single exposure.

1) Introduction and Background

In setting policies for the data rights of users of the Hubble Space Telescope, the STScI has traditionally made a clear distinction between imaging and spectroscopic observations. While successful HST users employing the observatory's cameras are awarded proprietary rights to the entire contents of the *field* covered by the exposures specified in their proposals, successful spectroscopic users are only assigned rights to the spectra of the individual *targets* explicitly called out in their proposals.

It has also been the case that HST users have only in special circumstances been permitted to specify the roll angle (or *ORIENT* in STScI parlance) at which their proposed observations are to be performed. These roll angles have instead been indirectly set by STScI when placing the observations on the overall the HST observing timeline.

The combination of these two policies has proven largely unproblematic for HST's spectrographic instruments, as these have been single-slit, single-object instruments. However, this is no longer the case with JWST, whose NIRSpec instrument centers around a purpose-built wide field Multi-Object Spectroscopic (MOS) functionality. The objective of this note is to explore the practical implications of NIRSpec MOS users needing to specify comprehensive and binding target lists that cater to a range in possible roll angle in their observing proposals.

It will be recalled that NIRSpec instrument achieves its multi-object capability by means of a so-called Micro-Shutter Array (MSA), made up of four quadrants of 365×171 individually addressable shutters arranged within a rectangular field of view spanning 3.6 arcmin × 3.6 arcmin = 13.3 arcmin^2 on the sky, of which 9.8 arcmin^2 is sampled by the four separate MSA quadrants. The large span of this field of view is by design intended to roughly match that of NIRCam.

The MSA's regular grid of fixed shutters of finite filling-factor, fed by an image of the sky displaying significant optical distortion arising in the instrument's relay optics and the JWST telescope proper, conspire to make NIRSpec a rather different and rather inflexible multi-object spectrograph compared to its ground-based predecessors. In particular, the only controls available to the user planning to observe a number of objects located within a given target field on the sky, is to adjust the JWST telescope fine pointing in pitch, yaw and roll within their allowable limits, while aiming to optimize the pattern of shutters to open such that the greatest number of high priority candidate targets line up with operational shutters that can be held open without their spectra overlapping on the NIRSpec detector array. This is a complex problem whose many subtle aspects are still being actively explored. To help NIRSpec users in this task, the STScI has in consultation with the NIRSpec Instrument Team been developing the so-called MSA Planning Tool (MPT), resident within the NIRSpec segment of the general JWST Astronomers Proposal Tool (APT).

It will also be recalled that the need to always keep the sunshade of the JWST observatory oriented near perpendicular to the Sun, not only restricts the instantaneous field of regard of the JWST telecope, but also strongly constrains the spacecraft roll angle that any accessable point on the sky can be viewed at on a given date. Specifying beforehand the roll angle at which a proposed observation is to be carried out at is therefore tantamount to locking down the date on, or near to which, the observation has to be performed. In the interest of maintaining flexibility in optimizing JWST's overall operational efficiency, the option of JWST users specifying roll angle will only be allowed when necessary for compelling scientific reasons. Instead, STScI will only inform the succesful JWST user of the roll angle at which a given observation will be executed once the observation in question has been scheduled and placed on the JWST observatory timeline.

This necessary and pragmatic approach, however, has an important consequence for prospective users of NIRSpec in its multi-object MSA mode. Without knowledge of the JWST roll angle, the NIRSpec user does not know the MSA's ultimate orientation on the sky, and therefore cannot run the MPT in earnest and identify which of the intended targets are covered by the MSA field of view and can be matched with slits before the Phase II information for the observation needs to be prepared and submitted late in the proposal implementation process – long after the initial proposal submission. The inescapable repercussion is that all NIRSpec MSA users will of necessity need to submit target lists that are deliberately over-booked by some suitable factor already at the proposal stage to assure that an adequately sized subset of the submitted target list commensurate with the scientific objectives of the proposal is available at any possible final roll angle.

The remainder of this note is concerned with quantifying how large this over-booking factor – here defined as the ratio between the average number of targets that need to be submitted and the average number that can be expected to be observed in a *single* NIRSpec MSA configuration – needs to be, given our current knowledge of the relevant characteristics and expected performance of the flight NIRSpec instrument.

There are three dominant factors entering the calculation of the required total over-booking factor. These reflect the three conditions needed for NIRSpec to be able to successfully obtain an MSA spectrum of a given target at a given telescope pointing: First the object needs to be captured within the field of view of the MSA. Next, the target needs to be adequately centered within a functional slit on the MSA. Lastly, there needs to be space available on the NIRSpec detector array to capture the dispersed spectrum of the target without incurring overlap with the spectra of other targets.

2) Capturing Targets Within the MSA Field of View

The first consequence of the late assignment of roll angle is the obvious one of the user needing to prepare for a range of possible orientations of the instrument field of view on the sky already at the proposal stage. Recall that the JWST telescope views the sky from behind the sun shade more or less perpendicular to the solar direction. Consequently, target fields close to the ecliptic are only accessible at a highly restricted range in roll angle twice a year, whereas target fields at the ecliptic poles are accessible at any roll angle over the course of the year. The range in roll angle that needs to be considered at the proposal stage therefore depends on the ecliptic latitude of the target field in question, and can be considerable.

The left panel of Figure 1 illustrates that if the MSA field of view is rotated by $\sim 90^\circ$ or more around its field center at a fixed right ascension and declination, the active shutters of the MSA sweep out an annulus with outer and inner radii of 155 arcsec and 22 arcsec. It follows that the lack of prior roll knowledge dictates that

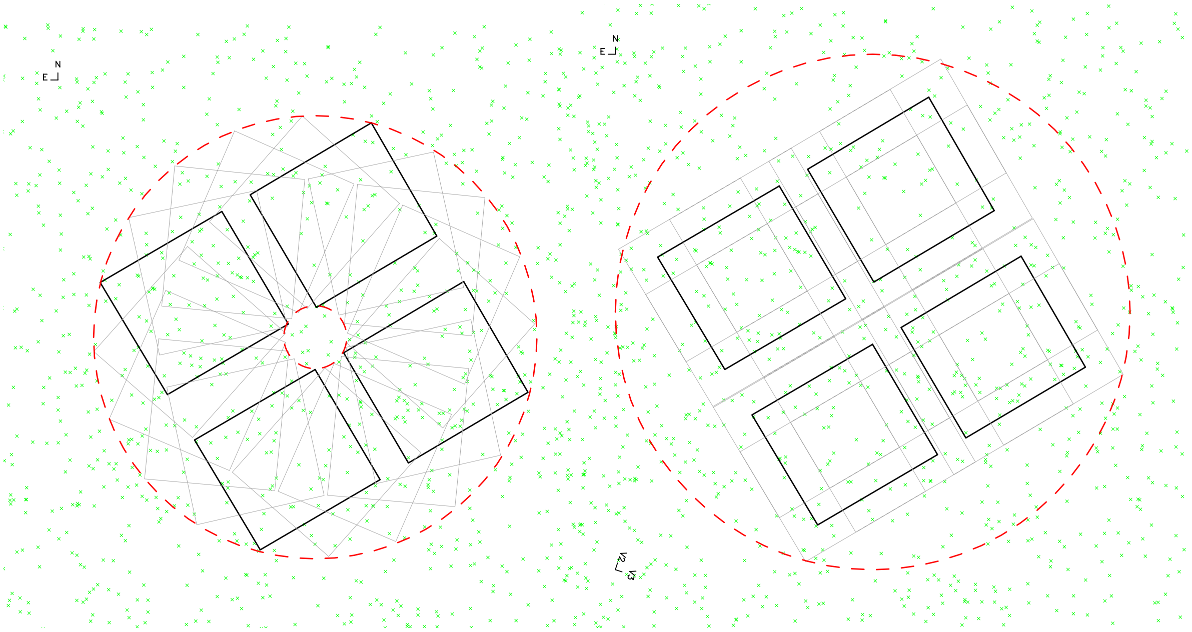


Figure 1: *Left:* If the NIRSpec MSA is rotated about its field center by 90° or more, the four MSA quadrants sweep out an annulus on the sky having an outer radius of 155 arcsec and an inner radius of 22 arcsec. *Right:* Allocating a slightly larger circular footprint with a radius of 3 arcmin to each MSA pointing will enable the field of view available at any final roll angle to be accessed in a contiguous manner, and thereby allow the flexibility required to optimize the observation in terms of high priority targets observed, access to reference stars, etc..

NIRSpec MSA survey programs will need to incorporate all targets of interest contained within this annulus in their respective proposals.

However, the tight annulus in Figure 1 is not workable in practice. Not only is it an impractical shape for all concerned, but it also needlessly locks down the telescope pointing in right ascension and declination, thereby hindering the optimization of the observation for the specified roll angle during the Phase II preparation. It also prevents the user from carrying out multiple observations of the field using different MSA configurations at slightly offset telecope pointings. As illustrated in the right panel of Figure 1, replacing the annular region with a circular zone having a slightly larger (and easy to remember) radius of 3.0 arcmin, allows just enough movement in pitch and yaw to allow the central area of the zone to be accessible in a contiguous fashion. This extended instrument footprint allows the observation to be optimized in terms of accessing the greatest number of high priority targets, the necessary number of reference stars for target acquisition, guide stars for guiding, etc. Assuming that the target density is uniform over the field, the probability that a given target listed in the observing proposal will be accessible in the NIRSpec MSA field of view in a single exposure at the roll angle eventually specified by the JWST planning system, is simply

$$p_{\Omega} = \frac{\Omega_{MSA}}{\Omega_{A}} = 0.347 \tag{1}$$

where $\Omega_{MSA} = 9.82$ arcmin^2 is the area on the sky covered by the four quadrants of the NIRSpec MSA, and $\Omega_{A} = 28.27$ arcmin^2 is the area spanned by the 3 arcmin radius roll-mandated extended footprint. The reciprocal of this probability then gives the required geometrical over-booking factor by which the full proposal target list covering all of Ω_A needs to exceed the subset target list covering only Ω_{MSA} that the user will have available to work with in carrying out the Phase II preparations for the observation in question

$$\Gamma_{\Omega} = p_{\Omega}^{-1} = \frac{\Omega_{A}}{\Omega_{MSA}} = 2.88 \tag{2}$$

Note that the perhaps surprisingly large magnitude of this ‘macroscopic’ geometrical over-booking factor is entirely dictated by the rectangular shape and spacing between the four quadrants of the MSA.

3) Capturing Targets Within Slits on the MSA

For a given JWST telescope pointing and orientation, the locations of the targets contained within the NIRSpec MSA field of view can be considered to be randomly distributed with respect to the MSA's shutter grid. There are two conditions that need to be fulfilled before a target can be assigned an MSA slit. The target needs to fall at an acceptable location within the open area of the shutter that it finds itself on, and the shutter needs to be functional.

3.1 The Open-area Filling-factor of the MSA and the Acceptance Zone

The MSA is manufactured to have uniform shutter pitches or facet widths of 105 μm in the dispersion (x) direction, and 204 μm in the spatial (y) direction. The central open areas of each shutter measures 76 μm in x and 175 μm in y. Due to optical distortion in the JWST telescope and the NIRSpec re-imaging optics, the projected sizes of the shutters on the sky vary by several percent in both dimensions across the MSA field of view. The average projection illustrated in Figure 2 has a 268 mas shutter pitch and 194 mas open width (74 mas wide bars) in the dispersion direction, and a 529 mas shutter pitch and 454 mas open height (75 mas wide bars) in the spatial direction. The mean open-area geometrical filling-factor of the MSA is therefore

$$p_F = (\frac{194 \text{ mas}}{268 \text{ mas}}) \times (\frac{454 \text{ mas}}{529 \text{ mas}}) = 0.621 \tag{3}$$

However, for most programs it is not sufficient that the target centroids are merely located within the open areas of their shutters. For point sources, the transmission of a single shutter drops off rapidly near the edges of the shutter open area, leading to a classical trade-off between the achievable spectrophotometric accuracy and how accurately the targets need to be centered within their slits. This trade-off is captured by the so-called *Acceptance Zone*, specifying the user-selectable area within the shutter open zones in which the MPT software will accept targets as sufficiently centered. Clearly, the smaller the Acceptance Zone, the smaller the effective filling-factor of the working area of the MSA becomes, and the fewer targets within the MSA field can be selected. If θ_x and θ_y denote the widths of the Acceptance Zone, the effective MSA filling-factor becomes

$$p_A = p_F\ p_Z \quad \text{where} \quad p_Z = (\frac{\theta_x}{194 \text{ mas}}) \times (\frac{\theta_y}{454 \text{ mas}}) \tag{4}$$

In what follows, it is useful to imagine the user having already identified the subset of n targets from the proposal catalog located within the four quadrants of the MSA at the roll angle assigned to the observation. Although the user can (and most likely will) attempt to use the MPT to fine-tune the JWST pointing, within allowable limits, so that one or two of the very highest priority targets are guaranteed to be well-centered within operational shutters, this has little impact on the overall statistics for larger values of n. Assuming the n targets to be randomly distributed over the MSA, the probability that any one of them falls within the Acceptance Zone of a shutter is then p_A, and the total number of targets, k, falling within the Acceptance Zones of shutters will obey a Binomial distribution $k \sim \mathcal{B}(p_A, n)$ with mean $E(k) = p_A n$ and variance $Var(k) = p_A(1 - p_A)n$.

The over-booking factor by which the list of n targets contained within the MSA field of view needs to be oversized due to the finite effective filling-factor of the MSA is then

$$\Gamma_A = \frac{n}{E(k)} = \frac{1}{p_A} = \frac{1}{p_F p_Z} = 1.61\ p_Z^{-1} \tag{5}$$

Depending on the users's choice of p_Z, this over-booking factor will in practice lie in the range $1.61 \leq \Gamma_A \leq 15.6$, corresponding to an Acceptance Zone spanning the full shutter open area to one having $\theta_x = 65$ mas and $\theta_y = 140$ mas ($p_Z = 0.108$), employing only the central one third of the open shutter width in the dispersion direction. Given this large uncertainty, the user-selectable parameter p_Z will in the following be explicitly called out in all expressions in which it appears.

3.2 The Impact of Failed Shutters and the Three-shutter Slitlet Concept

The previous section does not complete the story at the shutter level. The baseline is not to observe NIRSpec targets using single MSA shutters, but rather to employ three vertically adjacent shutters in a so-called slitlet to enable accurate local sky subtraction and dither out any detector imperfections by nodding the telescope and shifting the target between the three shutters between sub-exposures (Figure 2). It is therefore not enough for a given target to fall within the Acceptance Zone of a shutter, the shutter also needs to be both functional

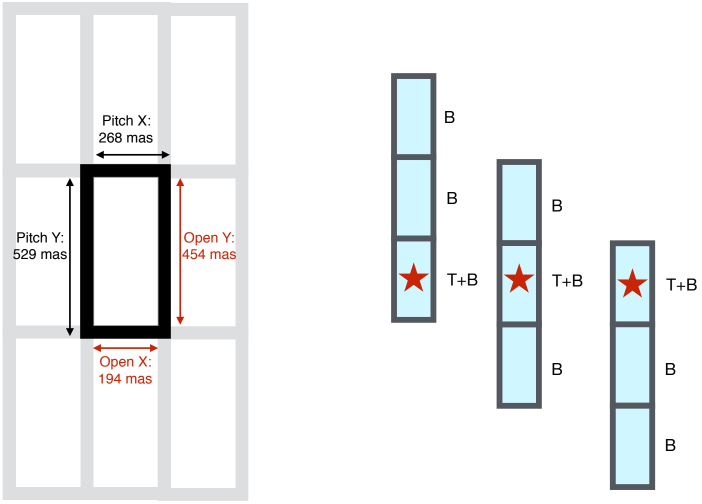


Figure 2: *Left:* Average dimensions of an MSA shutter projected onto the sky. *Right:* Illustration of the three shutter tall slitlet dithering and sky subtraction approach baselined for NIRSpec MSA observations.

and qualify as a so-called *Viable Slitlet*, that is, a shutter capable of serving as the *central* shutter of a complete 3-slitlet.

If the MSA were perfect, the number of Viable Slitlets contained in the four quadrants of the MSA would simply be

$$N_s = 4 \times 365 \times (171 - 2) = 246740 \tag{6}$$

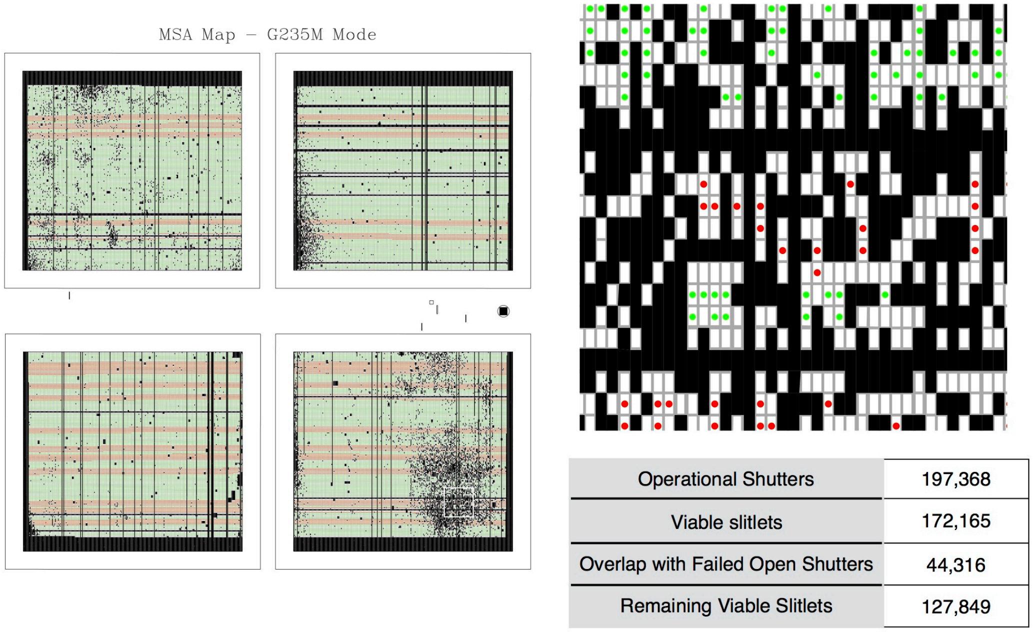
where the reduced row factor merely reflects that shutters in the top and bottom rows of each MSA quadrant cannot serve as the centers of complete 3-slitlets (although they can still serve as upper and lower shutters of such).

Alas, the NIRSpec flight MSA is far from perfect. As shown in Figures 3 and 4 it carries a considerable number of defects in the form of permanently failed open and failed closed shutters. Moreover, the MSA is intentionally slightly oversized, resulting in ∼12 rows and ∼10 columns along the extreme edges of the MSA field of view being vignetted by the field stop in the NIRSpec entrance plane. These unusable failed and vignetted shutters together conspire to effectively halve the total number of Viable Slitlets available over the MSA.

At the time of writing, the flight MSA possesses a total of 32450 (13%) non-operational failed closed shutters. As can be seen in the close-ups in Figures 3 and 4, many of these failed closed shutters, in addition to themselves, also prohibit neighboring shutters immediately above and below from qualifying as Viable Slitlets. These failed closed shutters, together with the 9% vignetted ones, serve to reduce the total number of Viable Slitlets in the MSA to $N_s = 172165$.

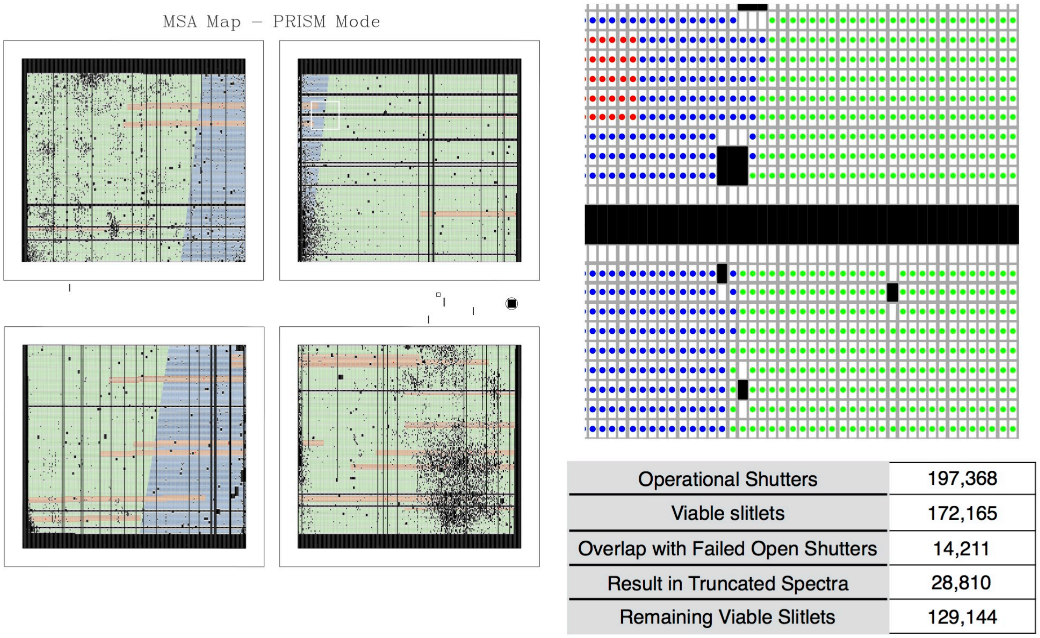
From this number needs to be further subtracted the impact of the permanently failed open shutters, of which there are presently 19 in the unvignetted portion of the MSA. Failed open shutters are in principle more onerous than failed closed ones, in that they are 'always on' and therefore leave dispersed single-shutter tall spectra of the sky background and any sources that happen to fall within them on the NIRSpec detector array. The approach to dealing with these failed open shutters in the present MPT is to avoid them, that is, to eliminate from use all Viable Slitlets that result in target spectra overlapping with those from the failed open shutters.

The number of Viable Slitlets affected by failed open shutters differs between NIRSpec's observing modes. The first-order $R = 1000$ grating spectra – flanked by their zero- and second-order counterparts on opposite sides – effectively span the entire width of the NIRSpec detector (see Figure 5 below), leading to large horizontal swaths of Viable Slitlets being excluded from use (Figure 3). This further reduces the total number of Viable Slitlets in this mode to $N_s = 127849$. Spectra taken with the $R = 100$ prism, on the other hand, have a finite length and display no higher or lower-order extensions, so in this case the failed open shutters do less damage. However, because of the shortness of the prism spectra, it makes sense to avoid slitlets resulting in truncated spectra falling partially within the gap between the two NIRSpec detector arrays (Figure 4). Doing so leaves a comparable total number of available Viable Slitlets as in the $R = 1000$ case: $N_s = 129144$.



Operational Shutters	197,368
Viable slitlets	172,165
Overlap with Failed Open Shutters	44,316
Remaining Viable Slitlets	127,849

Figure 3: Viable Slitlet map for the $R = 1000$ G235M grating. MSA shutters marked in green are viable as the central shutters of three shutter tall slitlets. Inoperable shutters that are either failed closed or vignetted by the NIRSpec Field Stop are indicated in black. Shutters marked in red are excluded due to their either being failed open (19 in all marked as solid squares), or lead to spectra that overlap with those of the failed open shutters (marked by red dots). Note that many of the unmarked shutters adjacent to failed closed shutters in white are still capable of being used, just not as the central shutters of three shutter slitlets.



Operational Shutters	197,368
Viable slitlets	172,165
Overlap with Failed Open Shutters	14,211
Result in Truncated Spectra	28,810
Remaining Viable Slitlets	129,144

Figure 4: As Figure 3, but for the $R = 100$ prism. Note the fewer number of red excluded shutters due to the short finite length of the prism spectra. However, in this case shutters marked in blue result in spectra truncated by the detector gap and are also excluded, altogether leading to a comparable number of remaining Viable Slitlets as in the $R = 1000$ case.

The probability that a shutter that a given target finds itself in happens to be a Viable Slitlet is then

$$p_S = \frac{N_s}{N_o} = \begin{cases} 0.512 & R = 1000 \text{ grating} \\ 0.517 & R = 100 \text{ prism} \end{cases} \tag{7}$$

where $N_o = 4 \times 365 \times 171 = 249660$ is the total number of shutters in the MSA.

The corresponding additional over-booking factor due to non-functional shutters is then:

$$\Gamma_S = p_S^{-1} = \frac{N_o}{N_s} \simeq 1.94 \tag{8}$$

It follows that the compounded probability that a target already within the MSA field of view falls within the Acceptance Zone of a Viable Slitet is

$$p_o = p_A \; p_S = p_F \; p_Z \; p_S \simeq 0.320 \; p_Z \tag{9}$$

The reciprocal over-booking factor due to all shutter-level geometrical factors is

$$\Gamma_\mu = p_o^{-1} = \Gamma_F \; \Gamma_Z \; \Gamma_S \simeq 3.13 \; p_Z^{-1} \tag{10}$$

This baseline 'microscopic' over-booking factor is in principle triggered by even the slightest uncertainty in the precise projection of the MSA onto the sky, and is applicable to any NIRSpec MSA observation that involves more than a single target.

4. Capturing the Spectra on the NIRSpec Detector

The third and final challenge faced by a candidate NIRSpec MSA target already located within the Acceptance Zone of a Viable Slitlet, is that there has to be space available on the detector array to accommodate its dispersed image without it interfering with the spectra of other accepted targets. It is the job of the MPT software to handle this important aspect when the user sets up the observation and configures the MSA. Briefly stated, for a fixed MSA projection on the sky, the task of the MPT is to first identify the n targets from the user-supplied candidate target list that fall within the MSA field of view, then determine which of the $k \leq n$ targets fall within the Acceptance Zones of Viable Slitlets, and finally determine which $m \leq k$ of these slitlets are capable of yielding non-overlapping spectra on the NIRSpec detector.

There are several ways to approach this task. The present MPT employs a priority-driven 'first come, first serve' algorithm, in which targets are processed in the order that they appear in the user-supplied target list, and each occupied slitlet is excluded from being commanded open if its spectrum conflicts with any of those placed before it. In this manner, the number of available Viable Slitlets remaining in the MSA slowly decreases as more targets are accommodated, and the MPT stops when either the end of the target list is reached, or all Viable Slitlets are accounted for. In the latter case, the MSA can be considered to be fully saturated, with the instrument having reached its maximum multiplexing capability for the observing mode in question. Illustrative simulations of this situation are shown in Figure 5. It is apparent that due to the different lengths and character of the prism and grating spectra, NIRSpec is capable of observing 3-4 times more targets simultaneously without overlap in $R = 100$ mode than in $R = 1000$ mode.[†] Moreover, as will be elaborated below, very large numbers of input targets are required to reach such fully saturated configurations.

Figure 6 and 7 display the outcomes of detailed Monte Carlo simulations of the functioning of the MPT, These so-called *Multiplexing Curves* plot the average number of targets that can be observed in a single exposure, m, as a function of the mean number of input candidate targets $n = \Sigma \; \Omega_{MSA}$ for an on-sky object density of Σ within the MSA field of view. Both simulations were carried out assuming the candidate targets to be randomly (i.e. uniformly) distributed over the MSA quadrants, and employ an open-area Acceptance Zone

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[†] The factor $\simeq 3.5$ difference in the maximum number of non-overlapping spectra that NIRSpec is able to record in $R = 100$ prism and $R = 1000$ grating modes is an issue for designing unbiased NIRSpec high-z galaxy surveys, and has led to an on-going debate within the NIRSpec GTO Team about the feasibility of carrying out $R = 1000$ observations using slit configurations optimized for the $R = 100$ prism, and accepting a modest amount of spectral overlap between the supposedly emission-line-dominated $R = 1000$ spectra. In the interest of not clouding the topic at hand with these deliberations, the focus of this paper is on the driving $R = 100$ case.

Figure 5: *Top:* Example of a fully saturated MSA mask configuration in the case of the $R = 100$ prism, showing the location and extent of 200 non-overlapping spectra on the two NIRSpec detector arrays. The spectra from failed open shutters are indicated in red. Note that by design, no prism spectrum falls within the gap between the two detector arrays, and that up to four prism spectra can be accommodated horizontally across the detector. *Bottom:* Equivalent fully saturated MSA mask for the $R = 1000$ G235M grating. The zero- and second-order spectra that bracket the primary first-order spectra are indicated in grey. Because the grating spectra span nearly the entire width of the detector, only 59 such spectra can be accommodated without overlap.

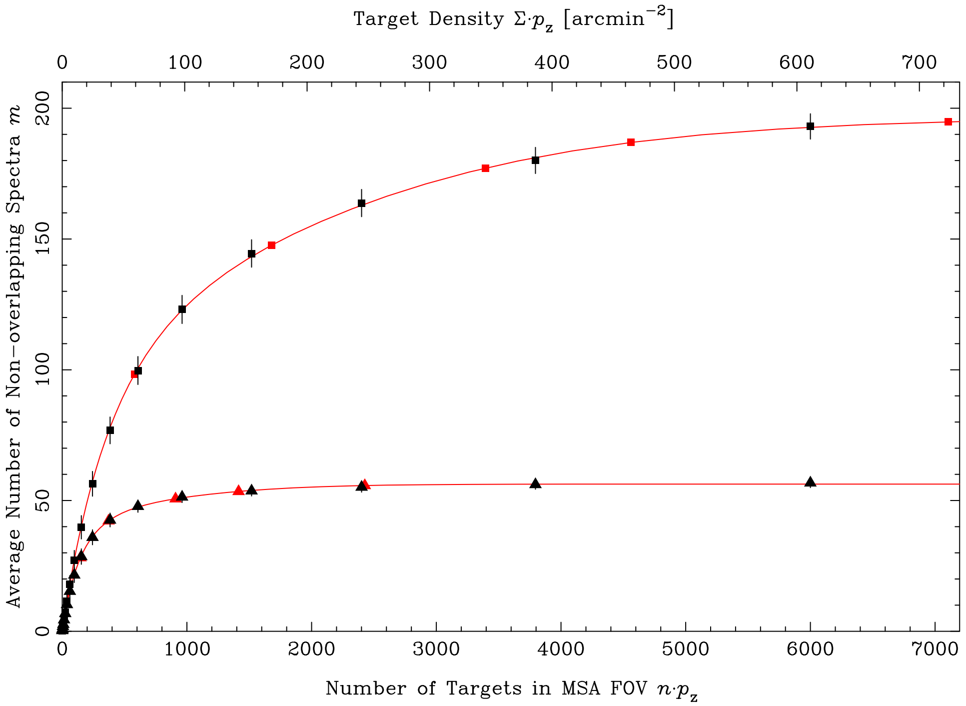

Figure 6: Results of the Monte Carlo simulations of the NIRSpec MPT showing the average number of non-overlapping spectra that can be placed on the detector as a function of the number (bottom scale) or surface density (top scale) of randomly located input candidate targets in the MSA field of view, employing the $R = 100$ prism (squares) and $R = 1000$ G235M grating (triangles). Each point shows the average and ± one standard deviation in the number of accommodated spectra m determined from 1000 random trials at each value of n. The red curves are the analytical model fits to the simulations described by equations (11) and (12).

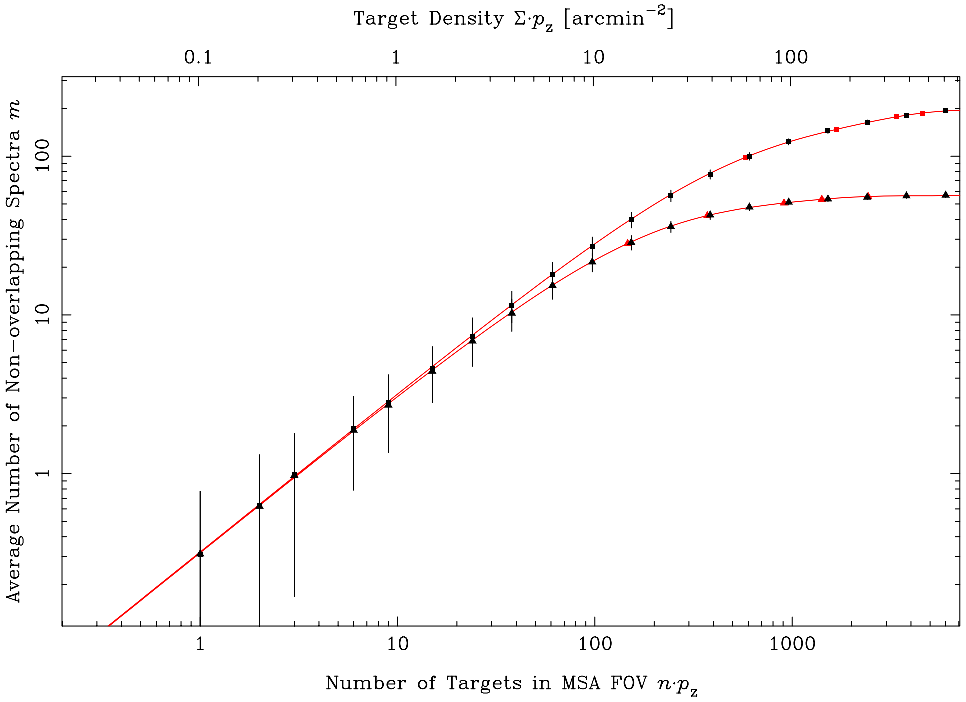

Figure 7: As Figure 5, but plotted on a log-log scale to bring out the linear range of the multiplexing curve at modest values of n well below saturation.

($p_z = 1$) in equation (9). Note also that the stochastic deviations away from the average values of m are modest at all values of input n, implying that the Multiplexing Curves provide a statisticaly adequate description of NIRSpec's multiplexing capabilities for most purposes.

The Multiplexing Curves are accurately reproduced by the simple analytical expression

$$m[p_o n] = m_s(1 - \exp(-\frac{p_o n\, \phi[p_o n]}{m_s})) \tag{11}$$

where p_o is given by equation (9) above, and

$$\phi[p_o n] = (\alpha_o + (1 - \alpha_o)\exp(-\beta_o p_o n)) \tag{12}$$

is a two-parameter soft step function that takes on the value $\phi = 1$ for small values of $p_o n$, and drops to $\phi = \alpha_o < 1$ for large $p_o n$.

The analytical fit overlaid in red in Figures 6 and 7 for the $R = 100$ prism has $p_o = 0.321$ (set beforehand by equation (9) with $p_z = 1$) and fitted parameters $m_s = 196.7$, $\alpha_o = 0.396$ and $\beta_o = 0.00316$. The fit for the $R = 1000$ G235M grating is $p_o = 0.318$, $m_s = 56.3$, $\alpha_o = 0.329$ and $\beta_o = 0.00598$. The latter fit is representative for all three $R = 1000$ gratings. A cardinal feature of equation (11) is the clean scaling in ordinate with p_o, which reflects the statistical independency between the processes of a target being assigned to a slitlet and there being space for its spectrum on the detector. The above values of m_s, α_o and β_o are therefore applicable for any choice of Acceptance Zone.

The flattening of the Multiplexing Curves at large values of n and Σ mirrors the increasing difficulty the MPT software faces when attempting to accommodate more targets on the NIRSpec detector array as the saturation limit $m \to m_s$ is approached in each mode. It is clear from equation (11) and Figure 7 that saturation effects are not an issue for NIRSpec MSA programs involving $n \ll m_s$ simultaneously observed targets. In this limit, equation (11) reduces to $m \simeq p_o n$, i.e. when there are too few spectra for overlap to be an issue, only the shutter-level selection discussed above – as captured by the parameter p_o – determines whether a given target in the MSA field of view can be observed or not.

The Multiplexing Curve equation (11) can be inverted to give n as a function of m

$$n[m] = -\frac{m_s}{p_o\, \phi[p_o n[m]]} \log(\frac{m_s - m}{m_s}) \tag{13}$$

with $\phi[p_o n[m]]$ given by (12). This equation is quickly solved exactly for n numerically through iteration.

The probability that a target in the MSA field of view and in a Viable Slitlet is accepted by the MPT based on spectral overlap considerations is then

$$p_M = \frac{m}{n[m]\, p_o} \tag{14}$$

where the extra p_o in the denominator corrects for the shutter-level censoring already being included in the ratio $m/n[m]$. The corresponding reciprocal over-booking factor due to multiplexing saturation effects alone is then

$$\Gamma_M = p_M^{-1} = \frac{n[m]}{m}\, p_o \tag{15}$$

It is clear that the multiplexing over-booking factor (15) is very different in nature from the two fixed and entirely geometrical over-booking factors discussed above, in that it depends on m, and thereby the density of candidate targets on the sky. This over-booking factor becomes significant for intermediate-to-high target density survey type programs aiming to exploit NIRSpec's multiplexing capabilities to the full.

Figure 8 plots Γ_M against m/m_s, the relative level of maximum multiplexing capacity that one aims to achieve. As expected, $\Gamma_M \simeq 1$ for small values of m/m_s, but shoots up to very large values as $m/m_s \to 1$ is approached. Table 1 lists the values of Γ_M, np_z and Σp_z required for m to reach 50%, 75%, 90%, 95%, and 99% of m_s. These benchmarks are also indicated in Figures 6, 7 and 8.

Table 1 reveals that even aiming to operate NIRSpec in MSA mode at a modest 50% of its full multiplexing capacity with the $R = 100$ prism ($m = 98$ simultaneous spectra), involves a multiplexing over-booking factor of $\Gamma_M = 1.89$, and the MPT software on average rejecting $(1 - p_M) = 1 - 1/\Gamma_M = 47\%$ of the input candidate

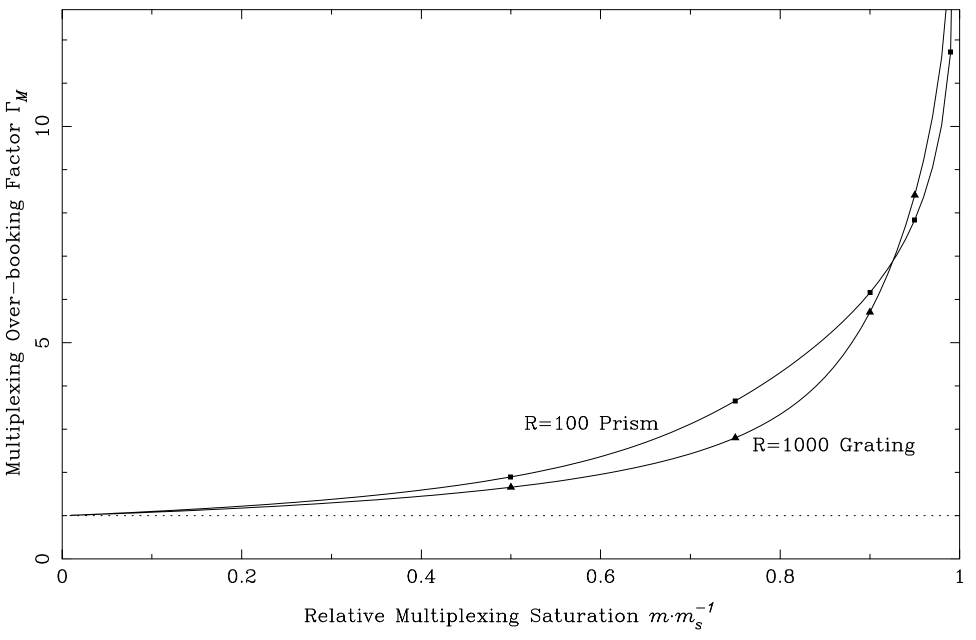

Figure 8: Over-booking factor due to multiplexing saturation effects, Γ_M for the $R = 100$ prism and $R = 1000$ grating modes, as a function of m/m_s, the relative efficiency of the maximum multiplexing capability one aims to achieve. The benchmark 50%, 75%, 90%, 95% and 99% multiplexing capacity cases listed in Table 1 are indicated by symbols on each curve.

Table 1: Benchmark High Multiplexing Cases

m :	$0.50m_s$	$0.75m_s$	$0.90m_s$	$0.95m_s$	$0.99m_s$
$R = 100$ Prism ($m_s = 196.7$):					
m	98.4	147.6	177.1	186.9	194.7
Γ_M	1.89	3.65	6.16	7.84	11.72
$n\,p_Z$	581	1678	3394	4561	7107
$\Sigma\,p_Z$ [arcmin^{-2}]	59.1	170.9	345.6	464.5	723.8
$R = 1000$ Grating ($m_s = 56.3$):					
m	28.1	42.2	50.7	53.5	55.7
Γ_M	1.65	2.80	5.70	8.41	13.84
$n\,p_Z$	146	371	909	1414	2426
$\Sigma\,p_Z$ [arcmin^{-2}]	14.9	37.9	92.6	144.0	247.1

targets otherwise situated in Viable Slitlets due to their spectra overlapping with those of previously placed targets. The equivalent rejection probability in $R = 1000$ grating mode ($\Gamma_M = 1.65$ for $m = 28$ simultaneous spectra) is a comparable 39%. The multiplexing over-booking factor becomes even larger as m/m_s approaches unity.

This rapid growth of the required size of the input target list at the highest multiplexing levels stems from the fact that NIRSpec is unable to move its remaining available slitlets to where the targets are, but instead must wait until a target appears in the MPT target list whose position happens to coincide with an available slitlet – thereby bypassing and rejecting an ever-increasing number of unsuitably located occupied slitlets between hits as the MPT algorithm progresses.

The benchmark cases listed in Table 1 reveal that achieving, 99% multiplexing efficiency in the workhorse $R = 100$ prism mode requires an on-sky target density greater than $\Sigma = 723.8\,p_Z^{-1}$ arcmin^{-2}, which is comparable to the *total* density of objects at all redshifts detected by HST in the HUDF. The NIRSpec MSA high redshift galaxy survey programs are expected to involve (primary) target densities well below this in the range of $\Sigma \simeq 50 - 300$ arcmin^{-2}, implying that the exposures of such programs will not reach detector saturation and the maximum multiplexing efficiency. The likely range in $R = 100$ prism multiplexing over-booking factor applicable for such programs is therefore $1.75 \leq \Gamma_M \leq 5.56$

Any NIRSpec MSA program involving a (primary) target density $\Sigma < 723.8\,p_Z^{-1}$ arcmin^{-2} can therefore be considered to be *target-limited*, in the sense that the total number of spectra such programs can expect to obtain is set by the number of candidate targets available on the sky and the number of separate MSA exposures taken, and not by the finite multiplexing capacity of NIRSpec. Such programs are therefore justified statistically in laying claim to *all* known sufficiently bright targets of the specified class of interest contained within the (roll-mandated extended) NIRSpec footprint as their initial proposal-level target list. In fact, not doing so is tantamount to wasting precious JWST observing time by not exploiting NIRSpec's multiplexing capabilities to the full and needlessly allowing available slitlets to remain un-used.

5. Discussion

It should be evident from the above analysis that NIRSpec in MSA mode – with its static pattern of slitlets and finite detector area – is entirely beholden to where Nature happens to place its targets within its field of view. As a result, the instrument is not particularly effective in utilizing its candidate targets for capturing spectra. The user needing to accommodate a range in roll angle from the onset, essentially moves confronting this fact out of the MPT software to front-and-center in the initial proposal stage. When further compounded by the large-scale geometrical impact of also needing to accommodate rotations of the NIRSpec field of view, this results in very large total over-booking factors given by the product of equations (2), (10) and (15)

$$\Gamma_T = \Gamma_\Omega\;\Gamma_\mu\;\Gamma_M = 9.0\;p_Z^{-1} - 50.9\;p_Z^{-1} \tag{16}$$

where we have set $1 \leq \Gamma_M \leq 5.56$ corresponding to anticipated on-sky target densities $\Sigma \leq 300$ arcmin^{-2}, and realistic values for the user-selectable size of the Acceptance Zone correspond to $0.1 \leq p_Z \leq 1$. It is again stressed that these over-booking factors refer to a *single* exposure and MSA configuration. There is obviously nothing to prevent the user from planning to obtain spectra of a larger number of objects in the same field by means of multiple exposures employing different MSA configurations optimized for slightly different telescope pointings. Nonetheless, it is clear that many such exposures will be required to obtain spectra of a significant fraction of all targets present in high density fields.

It is enlightening to examine the numbers involved in a concrete example in detail. Referring to Table 1 above, a program involving a relatively high total target density of $\Sigma = 170$ arcmin^{-2} will, with the choice $p_Z = 1$, still only achieve 75% multiplexing efficiency in $R = 100$ mode, corresponding to an average of $m = 0.75 m_s = 148$ non-overlapping prism spectra per MSA configuration. Reaching this number of spectra, however, requires the MSA field of view to contain a total of $n = \Sigma\Omega_{MSA} = m\,\Gamma_\mu\Gamma_M = 1678$ targets, which expanded to the roll-mandated extended NIRSpec footprint through equation (2), results in a total proposal-level target list containing $n_T = n\;\Gamma_\Omega = 4823$ targets, equivalent to a net target utilization efficiency of only $m/n_T = 3\%$. The small magnitude of this last number should be gauged against the two microscopic and macroscopic geometrical filling-factors captured by equations (1) and (9) alone guaranteeing that the NIRSpec MSA target utilization efficiency measured against the proposal catalog can never exceed $p_\mu\,p_\Omega = 11\%$. Furthermore, comparing this example with the 90% multiplexing efficiency case in Table 1 reveals that in order to achieve a mere $177 - 147 \simeq 30$ additional spectra on average, requires that there be a further $(3394 - 1678) \times 2.88 \simeq 4942$ candidate targets available on the sky that can be added to the proposal-stage target list – illustrating that securing the last spectra approaching the saturation limit requires a great many input targets indeed. Conversely, *not* attempting to secure these additional spectra in a given MSA configuration when the additional candidate targets are present in the MSA field, is also clearly wasteful.

The realization that the vast majority of NIRSpec MSA programs will be target-limited and in little danger of reaching maximum multiplexing saturation in $R = 100$ mode, forces us to revisit and refine the concept of the over-booking factor. Specifically, the clear distinction needs to be made between *mandatory* over-booking demanded by the need to compensate for the massive statistical pruning of the proposal-level target list that the last-minute roll-assignment policy and the NIRSpec MPT slit allocation process conspire to necessitate, as opposed to the perhaps more familiar concept of *unwarranted* over-booking in the sense of the observer intentionally or otherwise 'padding' the target list with objects that there in reality are no prospects of observing – and which should properly be made available to competing proposals. The latter sin is what first-use Guaranteed Time Observers are traditionally suspected of committing.

In the case of NIRSpec MSA programs, unwarranted over-booking – aside from through the generic crime of under-estimating exposure times – can in principle only occur at the very highest on-sky target densities for which there is the possibility of reaching full multiplexing saturation – defined in practice as the MPT algorithm stopping its process of assigning slits to targets before the end of the input target list is reached, due to it having run out of Viable Slitlets. Only in this situation can the candidate targets contained in the bottom portion of the target list that is never reached by the MPT be considered as extraneous padding (provided, of course, that these targets are not intended to be included in other separate exposures involving different pointings and MSA configurations). The 99% multiplexing case listed in Table 1 provides the benchmarks needed to allow the prospective NIRSpec MSA user to avert running into this unlikely situation (which, of course, is far more likely for high target density programs exclusively involving $R = 1000$ spectra).

A secondary consequence of most NIRSpec MSA programs being target-limited, is that such observations are bound to have an unknown number of unused viable slitlets left on the MSA after the primary targets have been allocated. In many cases there are likely to be interesting objects located in these slitlets of a type not explicitly called out in the original proposal. Leaving such slitlets closed would obviously be silly, so a flexible policy for how to deal with such cases is needed. One option is for the proposal to include a separate target list of such 'filler' targets at the proposal stage. However, for such a list to be effective, it would need to be very large, and therefore potentially over-booked at an unwarranted level.

While the over-booking factors arrived at in this paper are undeniably extremely sobering – and will very likely lead to suspicions of unwarranted prime target list padding – this remains the harsh reality that all NIRSpec MSA users – GTOs and GOs alike – will need to learn to accept. Pithily stated, there will aways need to be many more targets present in the MSA field of view than one can hope to observe in a single MSA configuration – but only rarely will there be enough available on the sky to completely fill the NIRSpec detector with spectra.

From a wider scientific perspective, the low efficiency in NIRSpec's ability to exploit its candidate targets would appear to be unfortunate in light of the limited lifetime expected of the JWST observatory, and the long turn-around time between observing cycles. This raises the notion of NIRSpec MSA proposals involving 'popular' fields such as the HUDF, GOODS, CANDELS, etc. releasing their 'unused' targets to competing programs, once the roll angles have been assigned and the MPT software has been run in earnest for all the MSA configurations and exposures involved. Clearly, an observing policy that is highly flexible, and at the same time highly disciplined, would be required to govern the issues raised by such an approach. Nonetheless, short of allowing NIRSpec MSA users to exceptionally dictate their roll angles in such high profile cases,[‡] this may well be the only way out of this quandary.

In any event, it is clear that NIRSpec MSA programs challenge the present paradigm for JWST users being awarded proprietary rights to spectroscopic targets. NIRSpec MSA users needing to submit heavily over-booked target lists in their proposals in order to use the instrument efficiently, is clearly not the same as requesting exclusive data rights and blocking these targets to competing proposals for the duration of the observing cycle, but rather reserving the right to 'use' these targets when running the MPT – and being content with only observing the small random subset of the submitted target list that the late assignment of roll angle and the awarded observing time happen to allow.

In weighing the issues raised in this note, one must not lose sight of the fact that in the final analysis, it is the absolute number of spectra of high redshift galaxies that NIRSpec will observe that is of scientific importance – and not the relative over-booking factors required to obtain them. In this regard, NIRSpec continues to hold great promise.

[‡] Such an approach would in any case only be possible in proposal Cycle 2 and following, since the necessary in-flight NIRSpec and JWST telecope optical distortion required for the MPT to align targets to slits at the required accuracy will not have been measured and verified on-orbit at the time of the Cycle 1 proposal deadline.